ME 20N: Haptics: Engineering Touch
Autumn 2017

Week 6: 2-Degree-of-Freedom Kinematics

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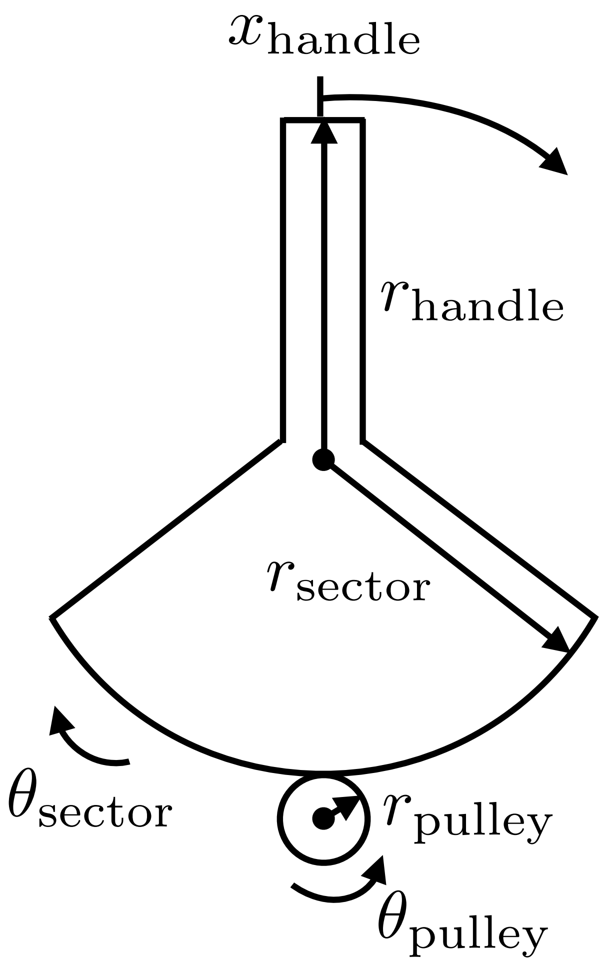
kinematics in multiple degrees of freedom

Hapkit kinematics (1-DOF)



$$r_{\text{pulley}}\theta_{\text{pulley}} = r_{\text{sector}}\theta_{\text{sector}}$$

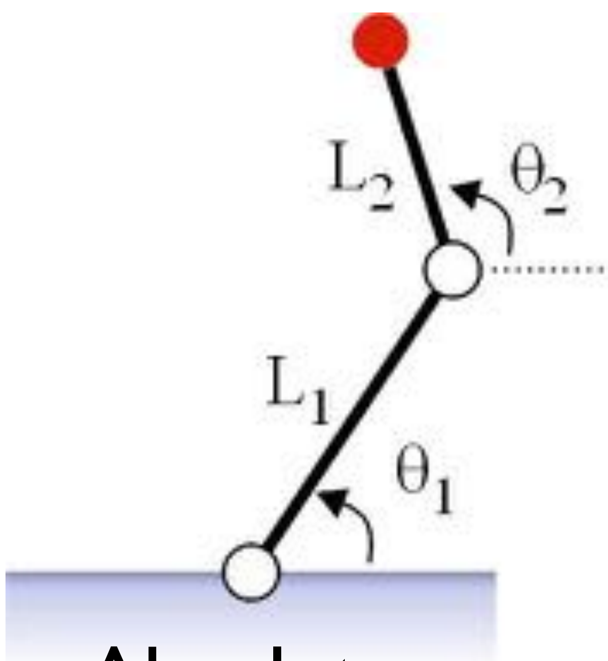
$$x_{\text{handle}} = r_{\text{handle}}\theta_{\text{sector}}$$

⬇

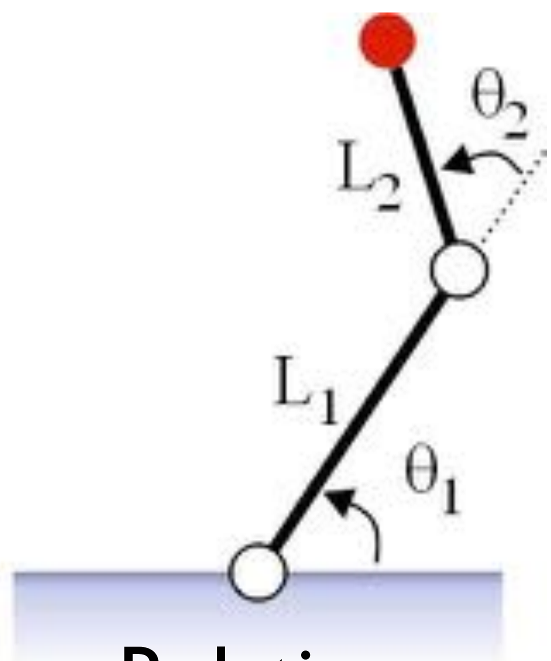
$$x_{\text{handle}} = \frac{r_{\text{handle}} r_{\text{pulley}}}{r_{\text{sector}}}\theta_{\text{pulley}}$$

joint variables

Be careful how you define joint positions

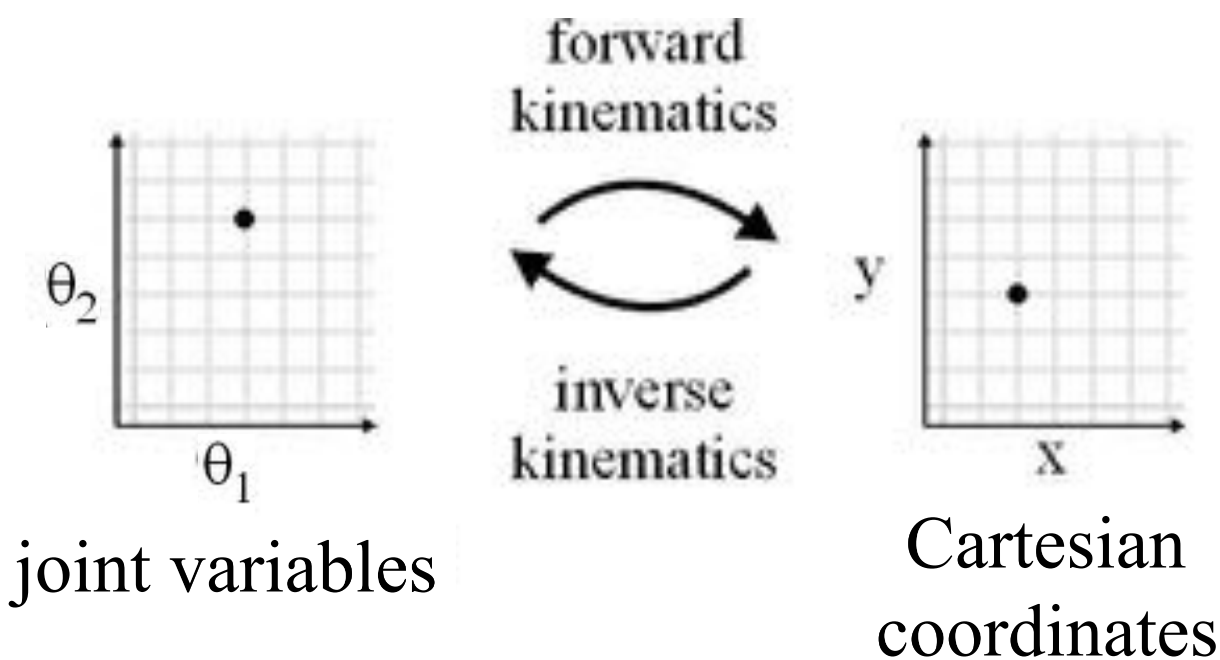


Absolute



Relative

forward kinematics for higher degrees of freedom



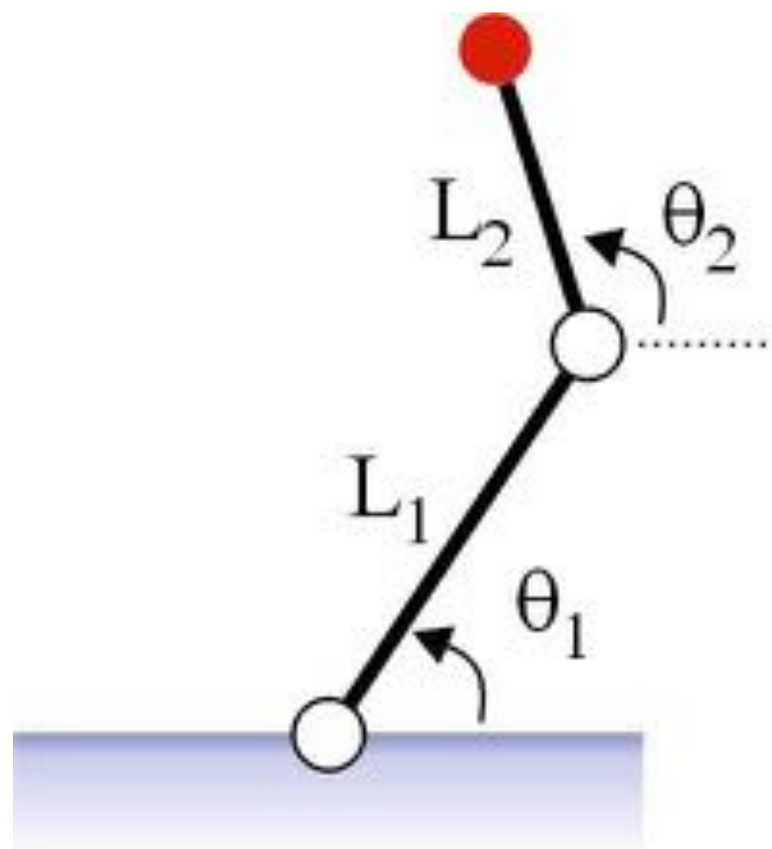
fwd kinematics: from joint angles, calculate endpoint position

serial structures

absolute forward kinematics

$x = L_1\cos(\theta_1) + L_2\cos(\theta_2)$

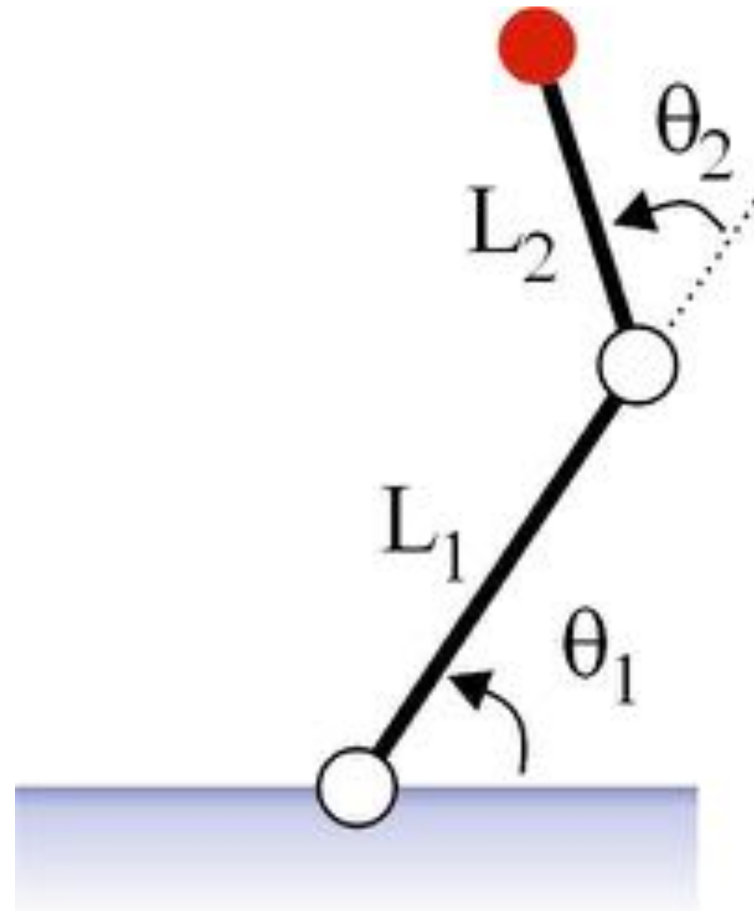
$y = L_1\sin(\theta_1) + L_2\sin(\theta_2)$



relative forward kinematics

$$x = L_1\cos(\theta_1) + L_2\cos(\theta_1+\theta_2)$$

$$y = L_1\sin(\theta_1) + L_2\sin(\theta_1+\theta_2)$$

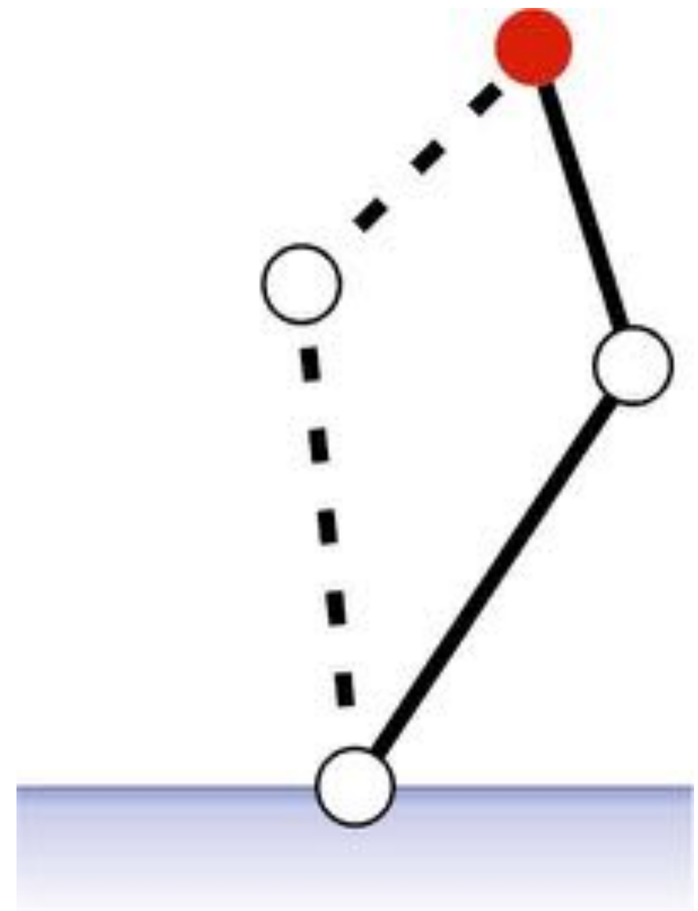


Inverse Kinematics

- Using the end-effector position, calculate the joint angles necessary to achieve that position
- There can be:
  - No solution (workspace issue)
  - One solution
  - More than one solution

example

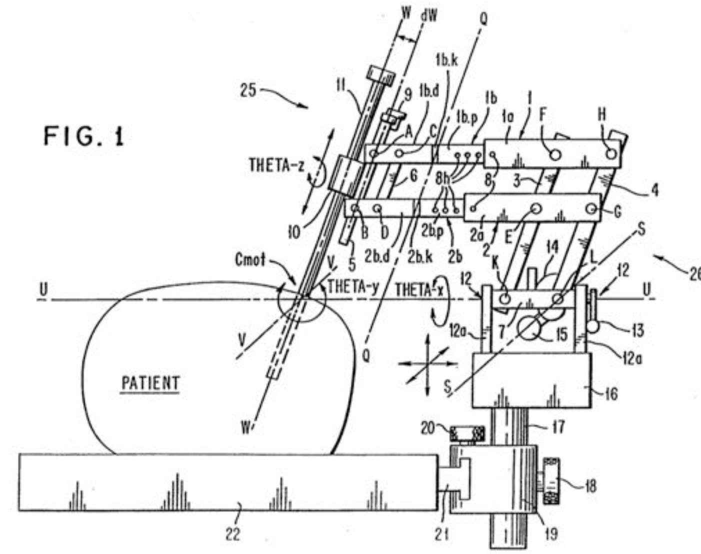
- Two possible solutions
- Our devices will be simple enough that you can just use geometry for inverse kinematics

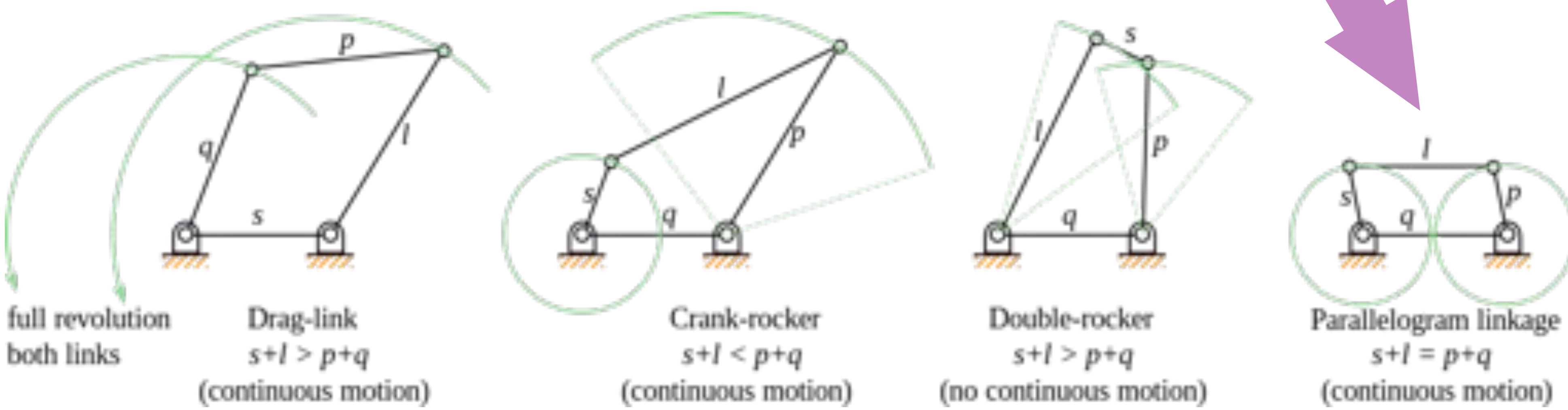


parallel structures

four-bar linkage

- commonly used 1-DOF mechanism
- relationship between input link angle and output link angle can be computed from geometry





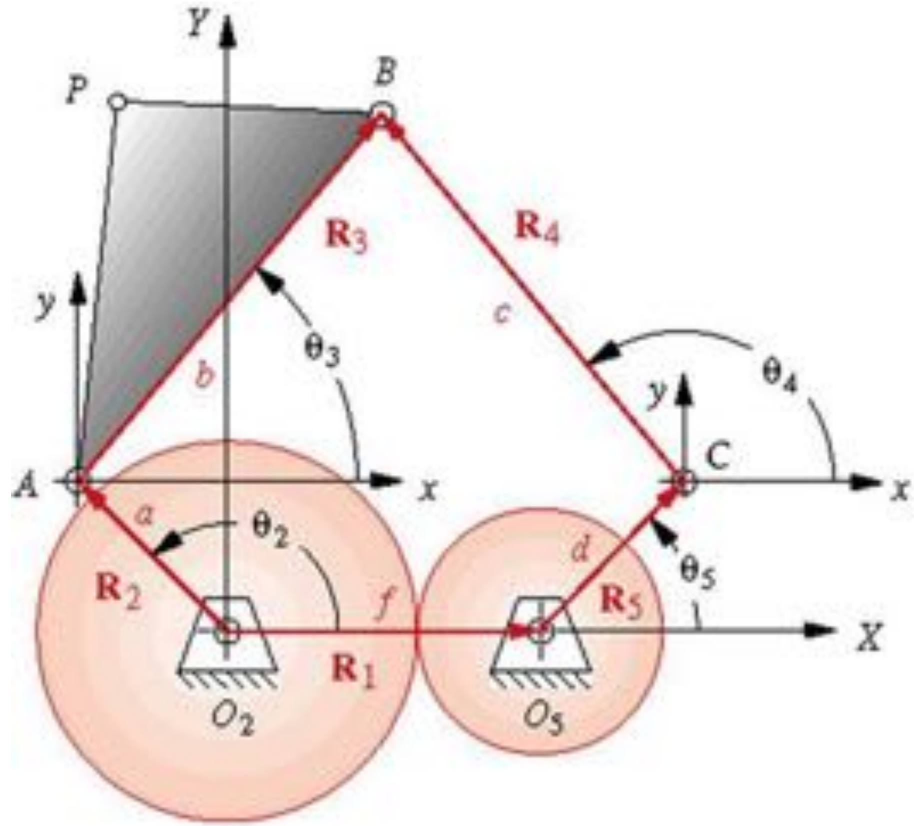
wikipedia.org

Types of four-bar linkages, *s* = shortest link, *l* = longest link

five-bar linkage

- commonly used 2-DOF mechanism
- relationship between input link angle and output link angle can be computed from geometry

example:

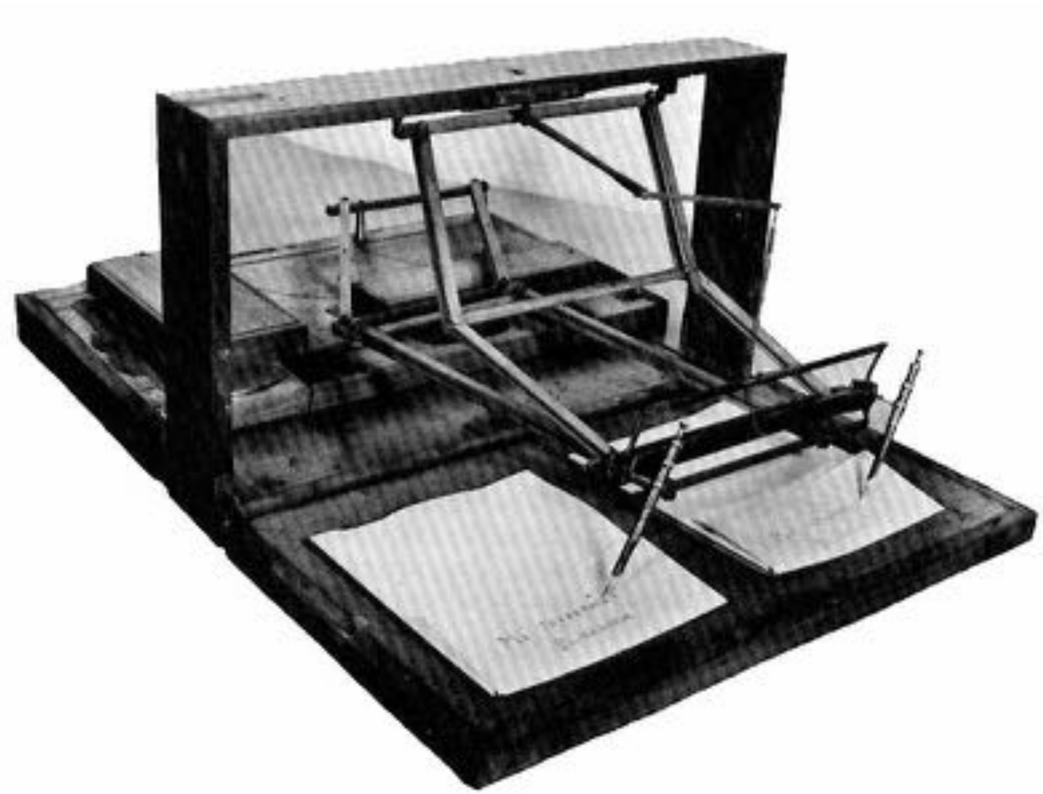


pantograph

Definition 1: a mechanical linkage connected in a manner based on parallelograms so that the movement of one pen, in tracing an image, produces identical movements in a second pen.

Definition 2: a kind of structure that can compress or extend like an accordion

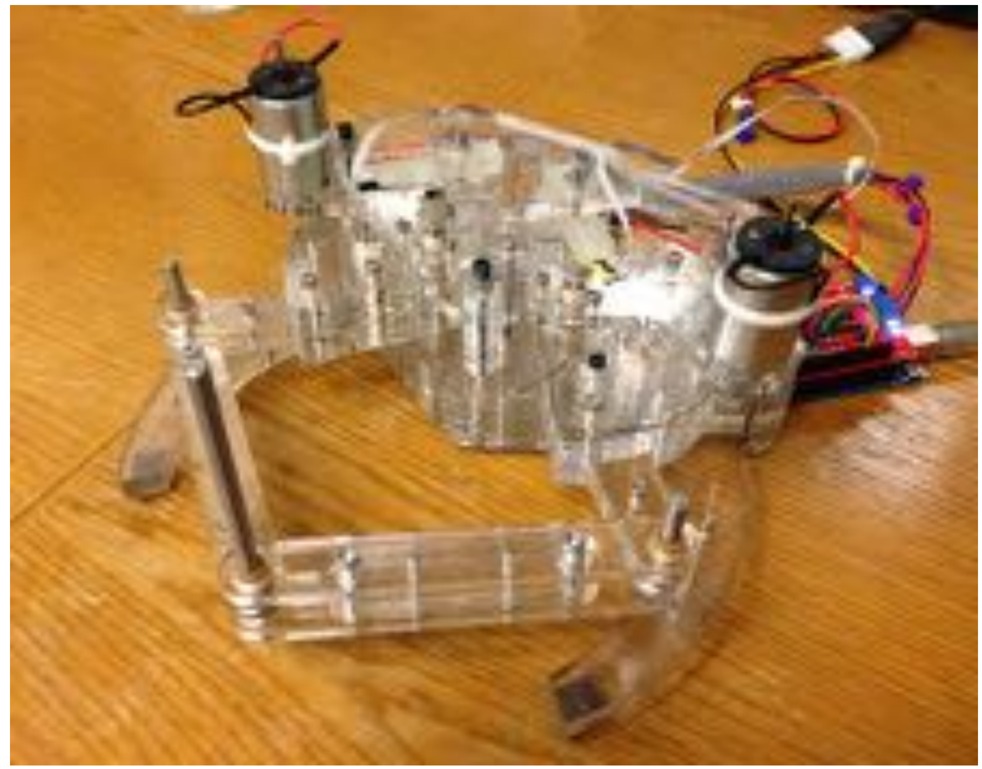
pantograph example



A Polygraph is a device that produces a copy of a piece of writing simultaneously with the creation of the original, using pens and ink. Famously used by Thomas Jefferson ~1805.

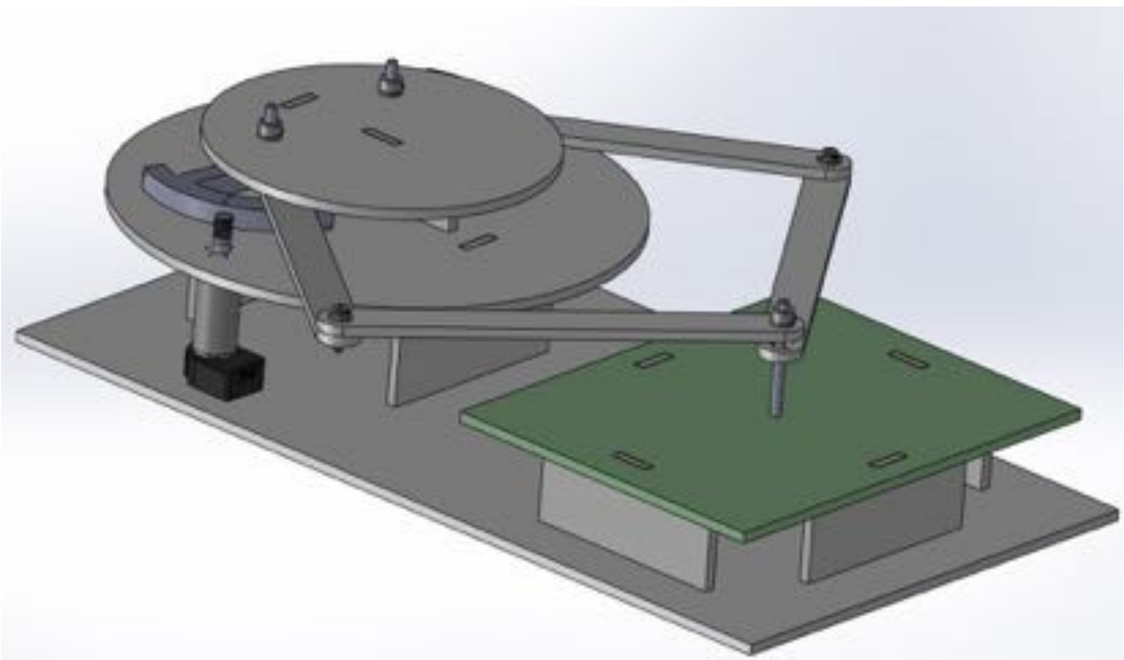
Typically uses a pantograph mechanism: a five-bar linkage with parallel bars such that motion at one point is reproduced at another point

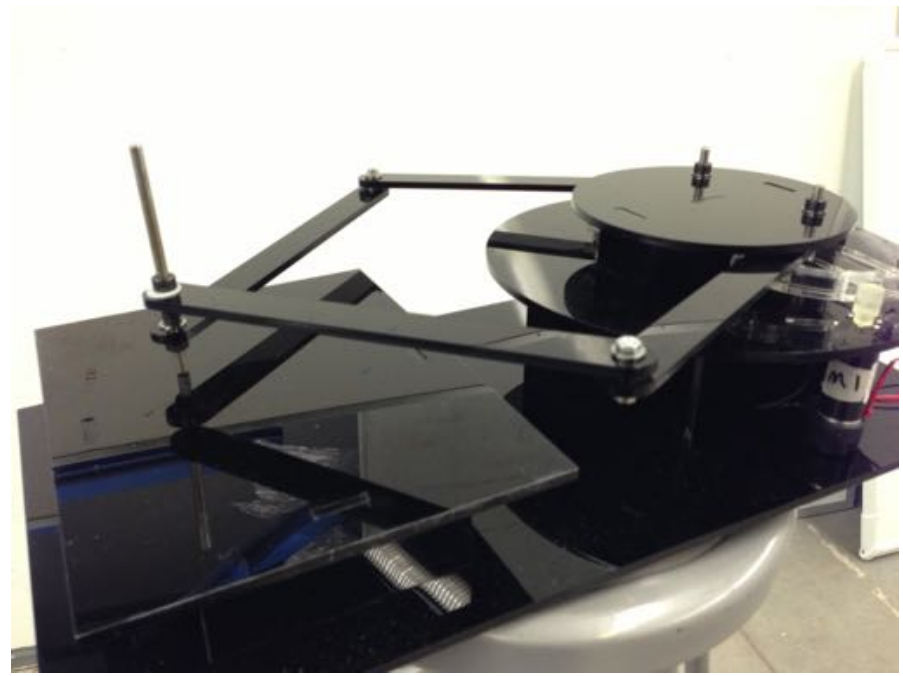
pantograph haptic device



Xiyang Yeh, ME 327 2012
http://charm.stanford.edu/ME327/Xiyang

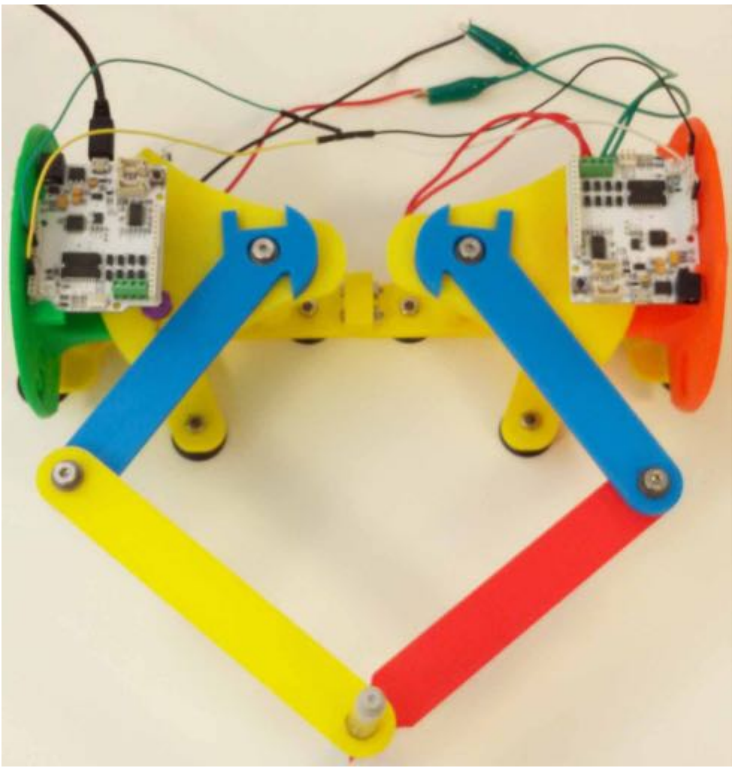
pantograph haptic device

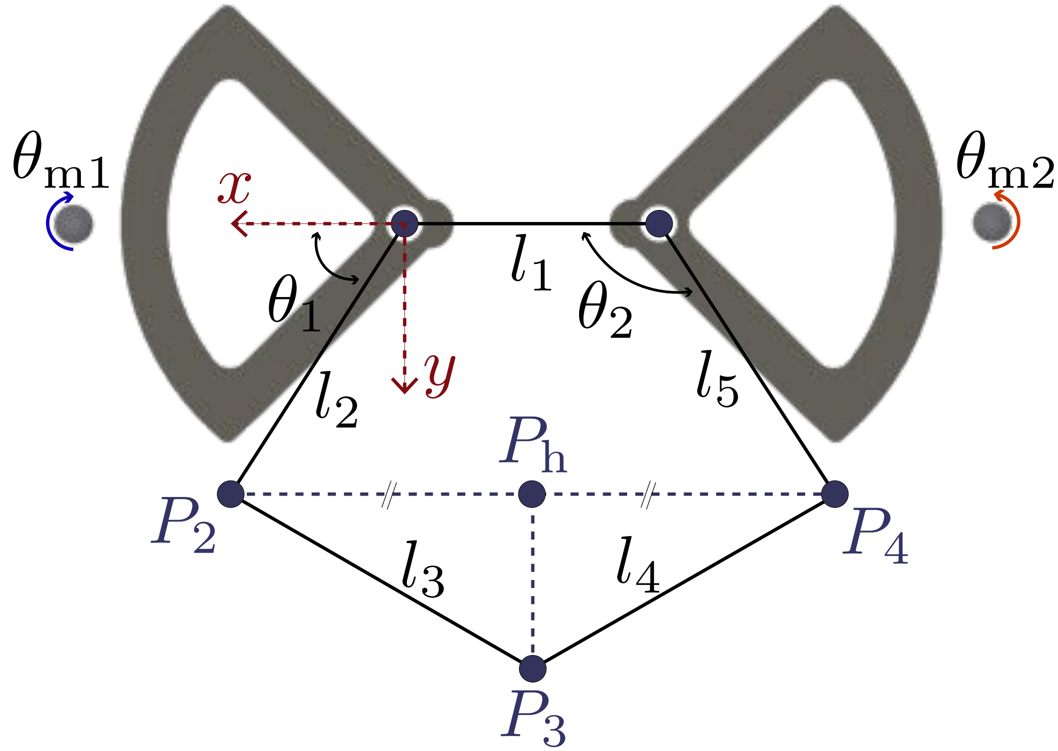




Sam Schorr and Jared Muirhead, ME 327 2012
http://charm.stanford.edu/ME327/JaredAndSam

pantograph haptic device

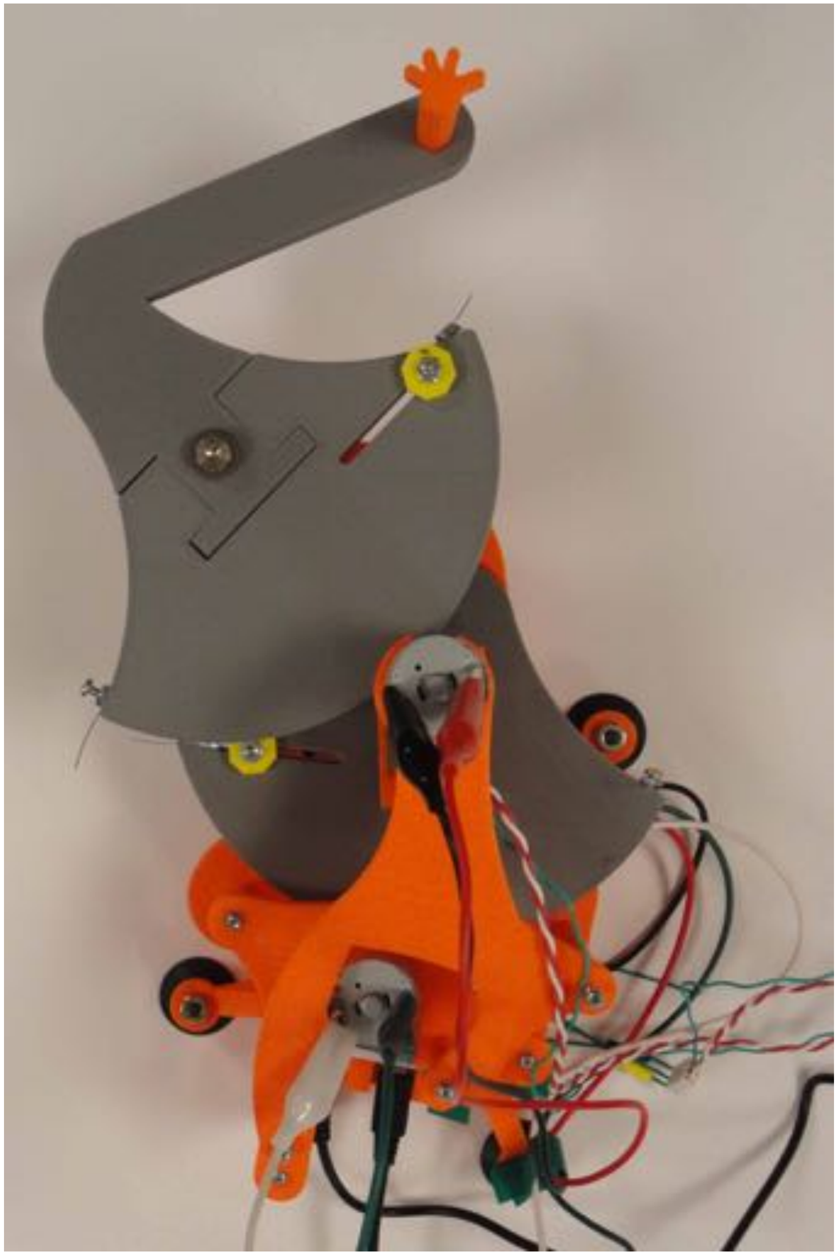


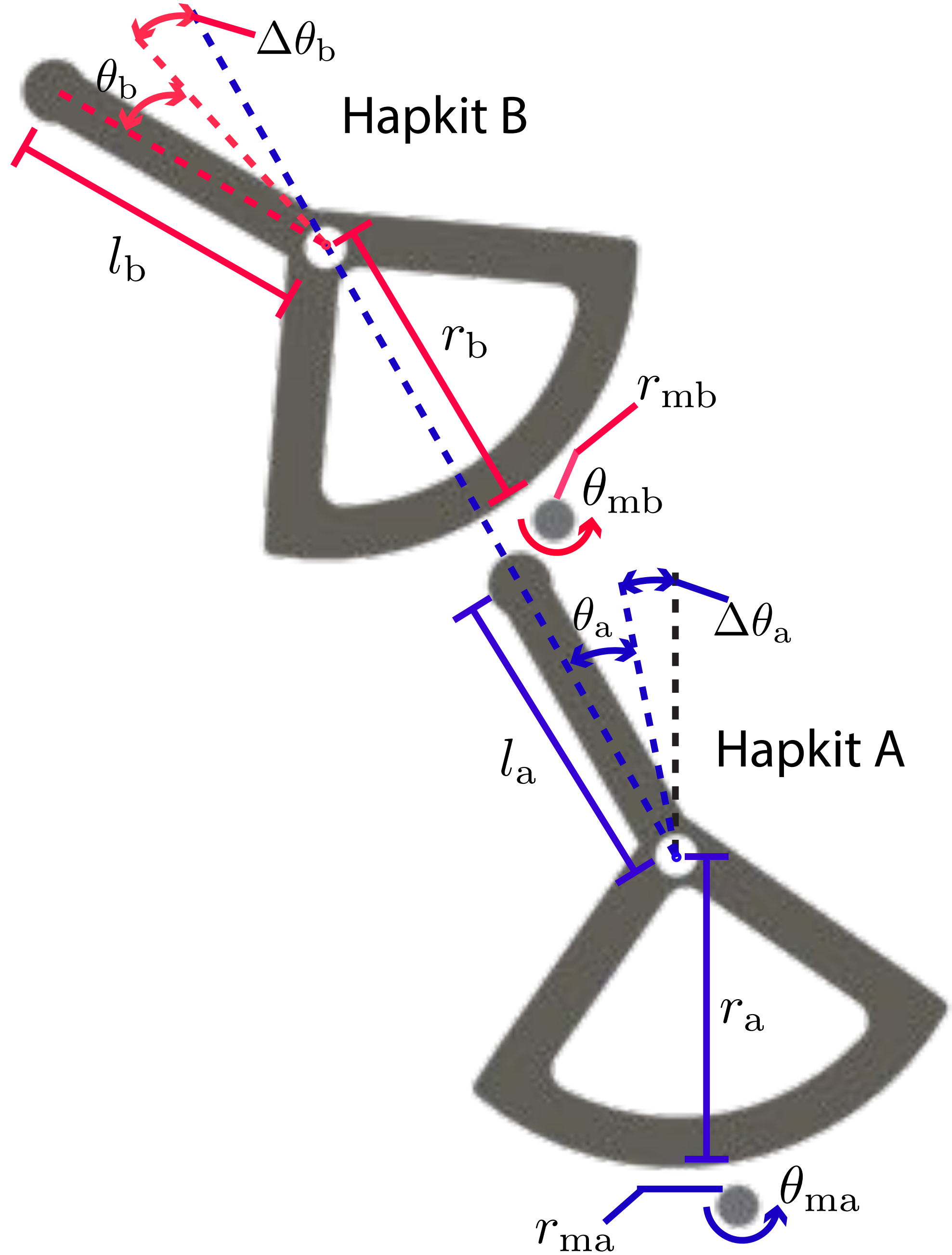


Melisa Orta Martinez et al., World Haptics Conference 2017
http://ieeexplore.ieee.org/document/7989891/

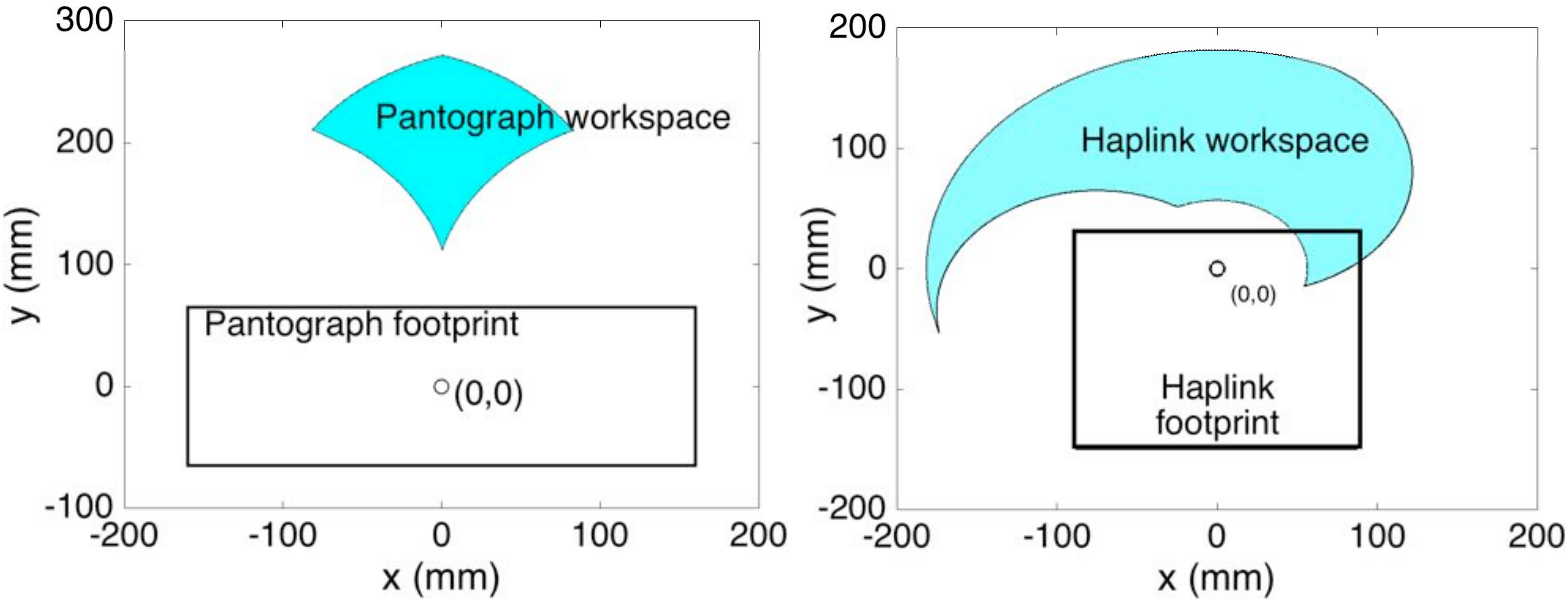
Haplink

Hapkit Graphkit Haplink



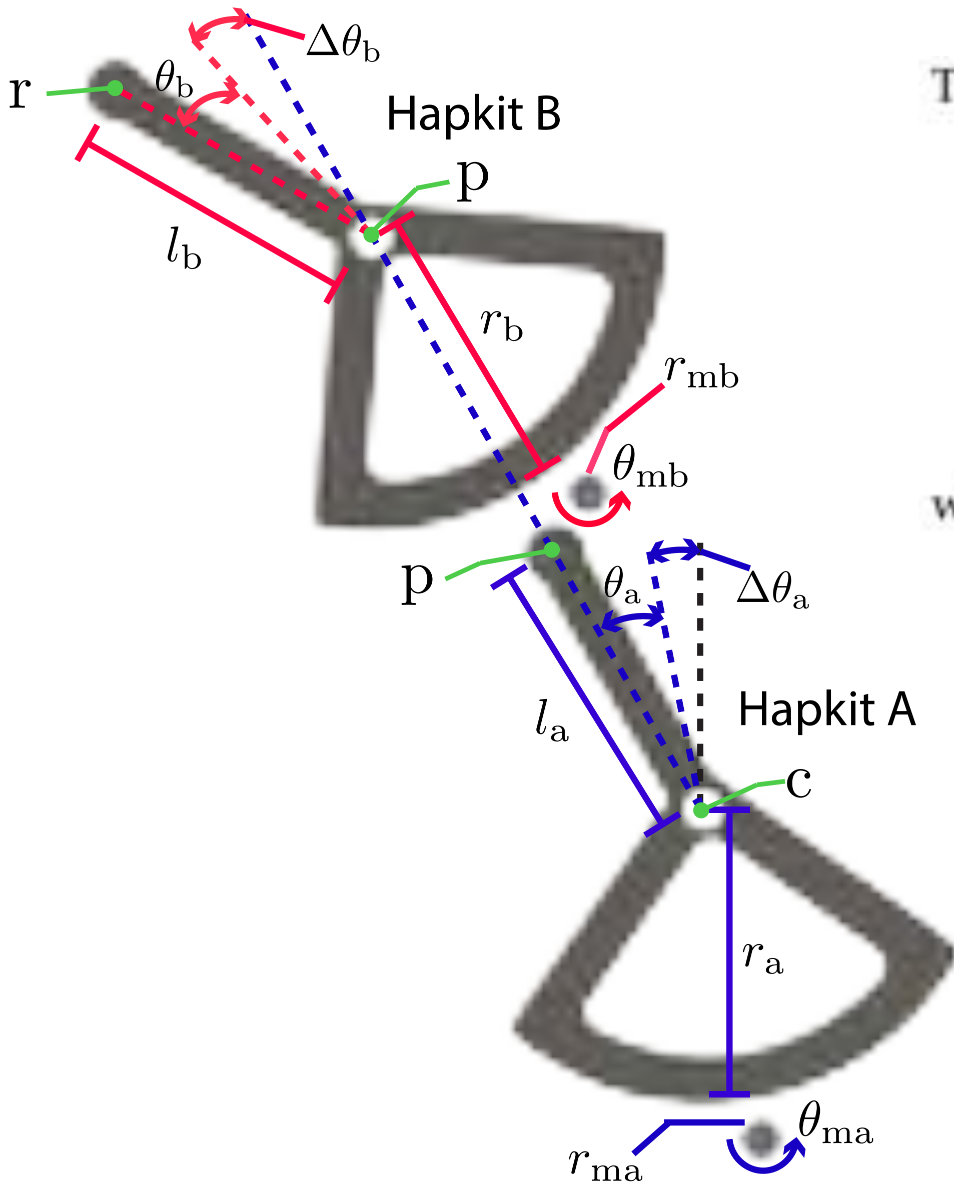

$\Delta\theta_b$
θ_b
Hapkit B
l_b
r_b
r_{mb}
θ_{mb}
θ_a
$\Delta\theta_a$
Hapkit A
l_a
r_a
θ_{ma}
r_{ma}

2-DOF Workspace



Q: What does a 1-DOF Hapkit workspace look like?

Kinematics



The forward kinematic equations are:

$$\begin{bmatrix} p_x \\ p_y \end{bmatrix} = \begin{bmatrix} -l_a \sin(\bar{\theta}_a) + c_x \\ l_a \cos(\bar{\theta}_a) + c_y \end{bmatrix}$$

$$\begin{bmatrix} r_x \\ r_y \end{bmatrix} = \begin{bmatrix} -l_b \sin(\bar{\theta}_a + \bar{\theta}_b) + p_x \\ l_b \cos(\bar{\theta}_a + \bar{\theta}_b) + p_y \end{bmatrix}$$

where:

$$\bar{\theta}_a = \theta_a + \Delta\theta_a$$

$$\bar{\theta}_b = \theta_b + \Delta\theta_b$$

$$\theta_{\mathrm{ma}} = -\frac{r_{\mathrm{a}}}{r_{\mathrm{ma}}}\theta_{\mathrm{a}}$$

$$\theta_{\mathrm{mb}} = -\frac{r_{\mathrm{b}}}{r_{\mathrm{mb}}}\theta_{\mathrm{b}}$$